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INFLUENCE OF THE OUTER MAGNETIC FIELD UPON THE SHAPE OF THE INTERFACE BETWEEN PLASMA FLUX AND THE PLANE DIPOLE CAVITY

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SUMMARY

The shape of the interface is determined between plasma flux with frozen-in magnetic field and the cavity filled by the magnetic field of a plane dipole. The cavity is found to be closed, and suggests a crescent with a hemispherical protuberance on the convex side, turned toward the flow of plasma.

If the external field is parallel to the velocity vector of the flow, the interface curves or bends inside in polar regions and in the posterior part near the equatorial plane. When the outer field is directed perpendicularly to the velocity, the hollows are absent at poles and in the posterior part, and the boundary of the tail is constituted by a circle.

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In the Chapman-Ferraro model the cold plasma is separated from the cavity filled by the magnetic field by a thin current layer [1 - 3]. The problem of the shape of this interface offers great interest in connection with the problem of solar wind interaction with the geomagnetic field. The determination of the exact shape of the interface is beset with great mathematical difficulties. Various two-dimensional models were considered in a series of works: the shape of the interface in case of plasma flow past an infinite conductor with current is determined approximately in [4], and the exact solution of the method of conformal transformation is given in [5, 6], the shape of the interface between plasma flow and the magnetic field of a plane dipole is found exactly in [7 - 9]. Methods for the solution of the corresponding three-dimensional problem have been worked out only lately; the self-consistent field method [10] and the method of technique of moments [11]. The three-dimensional problem on the shape of magnetosphere boundary at isotropic external pressure is resolved by the integral equation method in [12] and by the technique of moments in [13], and the corresponding two-dimensional problem - by the method of conformal transformation in [14, 15].

(*) VLIYANIYE VNESHNEGO MAGNITNOGO POLYA NA FORMU GRANITSY MEZHDU POTOKOM PLAZMY I POLOST'YU S PLOSKIM DIPOLEM.

In these models the interplanetary magnetic field, of 5γ intensity, is not taken into account [16, 17]; its pressure is negligibly small on the daytime side, where the main acting force is the dynamic pressure of the super-Alfvén flow. However, on the night side of the magnetosphere the interplanetary magnetic field may exert a substantial influence on the position and shape of the magnetosphere tail because of rapid drop in the value of the dipole field [18]. Because of the complexity of the problem, we shall consider a two-dimensional model in which only the dynamic pressure of the flux, the external field for the plasma outside the cavity, and the pressure of the magnetic field with the singularity of plane dipole type inside the cavity are taken into account. The gas-kinetic pressures inside and outside are supposed to be identical, for a rapid plasma diffusion through the interface takes place as a consequence of a series of instability mechanisms. The problem is resolved in the first approximation of the generalized self-consistent field method [10].

It is assumed that the plane dipole is directed perpendicularly to flow velocity, and two limiting cases of interplanetary field orientation are considered: along and perpendicularly to flow velocity. In the present model, we discounted certain factors exerting a substantial influence on the shape of the magnetosphere tail, and leading to the open magnetosphere model [19, 20] that finds experimental corroboration [21], and on account of this the results of our work fail to provide an answer to the question of the true shape of the magnetosphere tail.

STATEMENT OF THE PROBLEM. Assume that a plane magnetic dipole is situated at coordinate origin and directed along the axis y, and that the plasma velocity is directed along the axis x. The boundary condition is written in the form

$$|\mathbf{n}\times\mathbf{B}_i|^2-|\mathbf{n}\times\mathbf{B}_e|^2=8\pi p_0(\mathbf{n}\cdot\hat{\mathbf{v}})^2. \tag{1}$$

Here $\mathbf{B}_i$, $\mathbf{B}_e$ are respectively the inner and the outer field; p_0 is the dynamic pressure maximum; $\hat{\mathbf{n}}$ is the normal to the surface; $\hat{\mathbf{v}}$ is the unitary velocity vector. Moreover, we consider, as usual, that the normal component of the magnetic field at the interface is zero.

If the surface is given by the equation $F(r,\varphi)=r-R(\varphi)=0$, the projections of the normal, of the unitary velocity vector and of the magnetic field on unitary orts of a cylindrical system of coordinates will be presented in the following fashion:

$$\mathbf{n}=a\left\{1,-\frac{1}{r}\frac{dR}{d\varphi}\right\},\qquad \hat{\mathbf{v}}=\{\cos\varphi,\ -\sin\varphi\}.$$

$$\mathbf{B}_g=\frac{2M}{cr^2}\{\sin\varphi,\ \cos\varphi\},\qquad \mathbf{B}_0=B_0\{\cos\varphi,\ -\sin\varphi\}, \tag{2}$$

$$a=\left[1+\frac{1}{r^2}\left(\frac{dR}{d\varphi}\right)^2\right]^{-1/2}$$

Here $\mathbf{B}_g$ is the unperturbed field of the plane dipole; $\mathbf{B}_0$ is the outer magnetic field, parallel to the velocity. In the case when the outer magnetic field is perpendicular to flow velocity, we have

$$\mathbf{B}_0=B_0\{\sin\varphi,\ \cos\varphi\}. \tag{3}$$

If the shape of the surface is known, it is possible to find magnetic fields inside as well as outside. If to the contrary, the fields are known, the surface may be determined from (1). However, neither the fields B_e, B_i, nor the surface are known in (1). To overcome this difficulty we applied the method of self-consistent field proposed in [3]. Because of the presence of the outer field B_0 the generalization of this method is prerequisite. We shall consider in the first approximation that the interface is plane and we shall assume that the field B_1 induced by surface currents, constitutes the sum of the fields B_1 and B_2, whereupon the field B_1 compensates the dipole field outside, and the field B_2–the unperturbed outer field inside. The plane superficial current creates on either side of the plane magnetic fields, identical in magnitude, but opposite in direction. This is why, in the first approximation of the self-consistent field method we have

-- from the inside of the interface : $\mathbf{B}_i = \mathbf{B}' + \mathbf{B}_g + \mathbf{B}_0$,

— from the outside of it : $\mathbf{B}_e = -\mathbf{B}' + \mathbf{B}_g + \mathbf{B}_0$.

Inasmuch as

$$\mathbf{B}_1 + \mathbf{B}_g = 0, \quad \mathbf{B}_2 + \mathbf{B}_0 = 0,$$

we find

$$\mathbf{B}_i = 2\mathbf{B}_g, \qquad \mathbf{B}_e = 2\mathbf{B}_0. \tag{4}$$

Substituting relations (4) into (1), we obtain in the first approximation the following equation:

$$\frac{1}{r^4}\left(\cos\varphi - \frac{1}{r}\sin\varphi\frac{dr}{d\varphi}\right)^2 - \beta_0^2\left(\sin\varphi + \frac{1}{r}\cos\varphi\frac{dr}{d\varphi}\right)^2 = \left(\cos\varphi + \frac{1}{r}\sin\varphi\frac{dr}{d\varphi}\right)^2. \tag{5}$$

The unit of length $R_0 = (2M^2/\pi p_0 c^2)^{1/6}$ is introduced here; the dimensionless quantity $\beta_0 = B_0/(2\pi p_0)^{1/2}$ characterizes the outer field, which is considered to be oriented along the wind velocity.

Performing the substitution of variables $y = r^2$, $x = \mathrm{tg}\,\phi$, Eq.(5) will take the form

$$\frac{1}{y^2}\left(1 - \frac{1+x^2}{2}\frac{xy'}{y}\right)^2 - \beta_0^2\left(x + \frac{1+x^2}{2}\frac{y'}{y}\right)^2 = \left(1 + \frac{1+x^2}{2}\frac{xy'}{y}\right)^2.$$

In the case, when the field $\mathbf{B}_0$ is perpendicular to the velocity, we shall obtain instead of (6) the equation

$$\left(\frac{1}{y^2} - \beta_0^2\right)\left(1 - \frac{1+x^2}{2}\frac{xy'}{y}\right)^2 = \left(1 + \frac{1+x^2}{2}\frac{xy'}{y}\right)^2.$$

SHAPE OF THE INTERFACE WITH OUTER FIELD PARALLEL TO VELOCITY. Eq.(6) is sufficiently complex, thus making it practical to break the interface down to five regions (Fig.1). The first region (segment AB) spreads from the flow's stagnation point to the neutral point. The second region (BC) is the part of the interface between the neutral point and the center of the cavity. The third region (segment CD) is the portion between the center of the cavity and the point, at which the plasma flow hits directly the interface. The fourth region (DE) is the part of the tail subject to the action of the solar wind. The fifth region (segment EF) is the tail, protected from the action of the flow by the protruding fourth region. Eq.(6) may be simplified in each of the regions.

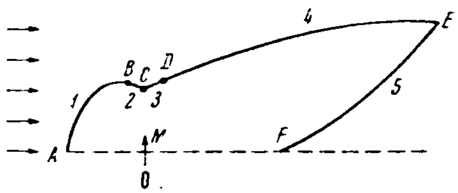

Fig.1

First Region. Here we may consider β_0^2 as a small parameter and search for the solution of (6) in the form $y = y_0 + \beta_0^2 y_1$. Then

$$y_0 = 1; \quad y_1 = -\frac{1}{2}\frac{\sqrt{1+x^2}}{x}\left[\ln(x+\sqrt{x^2+1}) - \frac{x}{\sqrt{1+x^2}}\right], \tag{8}$$

where we assumed during the determination of the constant in the solution that $y_1 = 0$ at $x = 0$, for at that point the outer field vector lacks the tangential component and exerts no pressure upon the surface.

The boundary of this region is the neutral point at which the surface (8) becomes parallel to the velocity. It is easy to see that at the neutral point $x = -2/\beta_0$. It is well known that in the first approximation of the self-consistent field method, the neutral point, in the absence of outer field, is located on the polar axis [22], whereas in subsequent approximations it is displaced in a direction toward the Sun by approximately 20° [10]. Therefore, the accounting for an outer field parallel to velocity leads to a supplementary shift of the neutral point in the same direction.

Second Region. Here the dynamic pressure of the flow is absent and Eq.(6) takes the form

$$\frac{1}{y}\left(1 - \frac{1+x^2}{2}\frac{xy'}{y}\right) = -\beta_0\left(x + \frac{1+x^2}{2}\frac{y'}{y}\right). \tag{9}$$

Taking into account that $x \gg 1$, it is practical to search for the solution in the form of a series by inverse powers of x :

$$y = \sum_{n=0}^{\infty} a_n x^{-n}, \tag{10}$$

where $a_1 = -2\beta_0 a_0^2$, $a_2 = -a_0 + 3\beta_0^2 a_0^3$, $a_3 = 2\beta_0 a_0^2(1 - {}^5/_3\beta_0^2 a_0^2)$. The coefficient a_0 may be found by joining the solutions in the neighboring regions, Third Region. The solution in the third region is given by formula (10), in which the sign before β_0 is reversed (here $x > 0$, so that in the second and third regions the interface is symmetrical relative to polar axis).

Fourth Region. In this region β_0^2 is also a small parameter; however, the zero solution

$$y_0 = \left(\frac{(1+x^2)^{1/2}+1}{x}\right)^2 \tag{11}$$

that is, the solution in the absence of the outer magnetic field differs from the zero solution in the first region; in connection with this the finding of y_1 becomes a more complex problem (the solution is sought for in the form $y = y_0 + \beta_0^2 y_1$).

Substituting (11) into (6), we obtain for y_1 the following equation:

$$y_1' + y_1 \frac{2\sqrt{1+x^2} - x^2}{x(1+x^2)} = \frac{(x^2 - \sqrt{1+x^2})^2(\sqrt{1+x^2}+1)^4}{2x^7(1+x^2)}. \tag{12}$$

The solution of (12) may be represented in the form

$$y_1 = \frac{\sqrt{1+x^2}(\sqrt{1+x^2}+1)}{8(\sqrt{1+x^2}-1)}\left[C + \frac{13x^2 - 18\sqrt{1+x^2}+17}{\sqrt{1+x^2}(\sqrt{1+x^2}-1)^2} + \ln\frac{(\sqrt{1+x^2}-1)^{17}}{x(1+x^2)^8}\right]. \tag{13}$$

For $x \gg 1$ the value of y_1 is

$$y_1 \approx \frac{x}{8}\left(C + \frac{13}{x}\right);$$

joining the solutions in neighboring regions, we obtain $C = -2\beta_0 \ln\frac{2}{\beta_0} - \frac{13}{2}\beta_0$. For $x \ll 1$

$$y_0 \approx \frac{4}{x^2} + 2 - \frac{x^2}{4}, \quad y_1 \approx -\frac{2}{x^6} + \frac{17\ln 2 - 33\ln x}{2x^2}. \tag{14}$$

Let us find the point, at which the tangent to the interface is parallel to flow velocity; assuming that at that point $x_0 < 1$, and utilizing (14), we arrive at the approximate equation

$$x^8 - \beta_0^2\left(34\ln 2 + 33 + 66\ln\frac{1}{x}\right)x^4 - 24\beta_0^2 = 0. \tag{15}$$

If β_0 is very small, we have approximately $x_0 \approx \sqrt[8]{24\beta_0^2}$. Typical for the solar wind is the value $\beta_0 = 0.1$, and for such a β_0 the rejected middle term in (15) may result to be essential. However, determining graphically the root of Eq.(15), we may find the assurance that the accounting of this term changes very little the value of the root found by the approximate formula.

Fifth Region. Here the dynamic pressure does not act upon the surface and Eq.(6) thus takes the form

$$\frac{1}{y}\left(1-\frac{1+x^2}{2}\frac{xy'}{y}\right)=\beta_0\left(x+\frac{1+x^2}{2}\frac{y'}{y}\right), \tag{16}$$

or

$$\frac{y'}{y}=2\frac{1-\beta_0 xy}{(1+x^2)(\beta_0 y+x)}.$$

We shall search for the solution of this equation by the method of consecutive approximations, selecting for the zero approximation the quantity $y|_{x=x_0}=y_0$ at the initial point of the fifth region, $x = x_0$.

The formal solution will be written in the form

$$y=y_0\exp\left[-2\int\limits_x^{x_0}\frac{1-\beta_0 xy}{(1+x^2)(\beta_0 y+x)}dx\right]$$

whence in the first approximation

$$y_1=y_0\exp\left[-2\int\limits_x^{x_0}\frac{1-\beta_0 y_0 x}{(x+\beta_0 y_0)(1+x^2)}dx\right]$$

or

$$y_1=y_0\frac{(x+\beta_0 y_0)^2}{(x_0+\beta_0 y_0)^2}\frac{1+x_0^2}{1+x^2}. \tag{17}$$

SHAPE OF THE INTERFACE WHEN THE OUTER FIELD IS PERPENDICULAR TO VELOCITY.

First Region. As in the preceding case we must consider β_0^2 as a small parameter. It may be seen that the unique solution of Eq.(7), not having a singularity on the polar axis, is

$$y_0=1, \quad y_1=-\frac{1}{2}. \tag{18}$$

The Second and the Third Regions are absent.

Fourth Region. We shall seek the solution in the form of series by the small parameter β_0^2. Substituting the sero solution (11) into (7), we obtain

../..

$$y_1' + y_1 \frac{2\sqrt{1+x^2} - x^2}{x(1+x^2)} = \frac{1}{2} \frac{(1+\sqrt{1+x^2})^6}{x^5(1+x^2)}, \tag{19}$$

of which the solution is

$$y_1 = \frac{\sqrt{1+x^2}(\sqrt{1+x^2}+1)}{2(\sqrt{1+x^2}-1)} \left[C + \frac{1-5\sqrt{1+x^2}}{\sqrt{1+x^2}(\sqrt{1+x^2}-1)} + \ln \frac{(1+x^2)^2}{(\sqrt{1+x^2}-1)^4} \right]. \tag{20}$$

As $x \to \infty$, the solution must be finite; this is why we must consider $C = 0$. The solutions of first order relative to $\beta_0{}^2$ in the first and fourth regions then have a discontinuity of first kind on the polar axis, result which is by no means unexpected. It is well known that in the first approximation of the self-consistent field method there are points, at which the solutions of Eq.(1) do not exist if the exact value of the magnetic field is substituted by an approximate one [10]. The following approximations must liquidate this discontinuity. If the magnetic field $\mathbf{B}_0$ is oriented strictly perpendicularly to the flow, the discontinuity vanishes too and the constant C is not zero.

The second singularity of the case considered, when the velocity vectors and the outer field are perpendicular, is the absence of such a point in the fourth region, where the surface is parallel to the velocity. Indeed, had such a point existed, the dynamic pressure would be absent at it, both of the flow and of the outer magnetic field, since the velocity is parallel to the tangent to the surface, while the outer magnetic field has no tangential component. As a consequence, the terms

$$1 - \frac{(1+x^2)}{2} \frac{xy'}{y} \quad \text{and} \quad 1 + \frac{(1+x^2)}{2} \frac{xy'}{y},$$

of (7), would become zero concomitantly, which is impossible.

Fifth Region. The dynamic pressure of the flow is absent and Eq.(7) takes the simple form

$$\left(\frac{1}{y^2} - \beta_0^2\right)\left(1 - \frac{1+x^2}{2} \frac{xy'}{y}\right)^2 = 0. \tag{21}$$

It is seen that the unique solution having any physical sense is the circumference

$$y = 1/\beta_0. \tag{22}$$

Therefore, the boundary of the fourth region is the point of intersection with the circumference (22).

We represented in Fig.2 the upper half of the interface at $\beta_0 = 0.1$. The solid curve corresponds to the case of the outer field parallel to the velocity, and the dashed curve — to the case of the outer field perpendicular to the velocity. The unit of length is the distance to the interface at flow stagnation point.

At $\beta_0 = 0.1$, Eq.(15) takes the form

$$x^8 - \left(0{,}57 + 0{,}66 \ln \frac{1}{x}\right) x^4 = 0{,}24. \tag{23}$$

If we reject the second term from the left, $x_0 = 0.84$. The graphical solution of (23) gives $x_0 = 0.96$. This result was utilized during the construction of the curve in Fig.2.

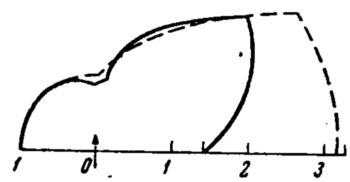

Fig.2

It should be noted that the results obtained in the present work are not applicable for the investigation of the structure of the magnetosphere tail, for in reality, besides the magnetic and dynamic pressures other forces act in the tail [19, 20], of which the nature is still not quite well ascertained. Moreover, the small value of the distance, over which the tail of the interface closes, is found to be in contradiction with the estimate of cavity length behind the magnetosphere, ($\sim 100 R_E$), into which the solar plasma does not reach because of the high Mach number [19]. Experimental measurements of magnetic field intensity in the magnetosphere tail have shown that the interface does not close at least up to $30\ R_E$, and the magnetic field is directed radially and changes sign at transition through the neutral layer situated in the equatorial plane [21].

APPENDIX

RIGOROUS SOLUTION OF EQ.(7). This equation may be represented as follows:

$$\frac{y'}{y}\,\frac{\sqrt{1-\beta_0^2 y^2}+\delta y}{\sqrt{1-\beta_0^2 y^2}-\delta y} = \frac{2}{x(1+x^2)},$$

where $\delta = 1$ at the forward part of the interface, $\delta = -1$ at the part of the tail subject to dynamic pressure of the flow, and $\delta = 0$ in the part of the tail not directly hit by the flow. The rigorous solution of this equation has the form

$$2 \ln \frac{x}{\sqrt{1+x^2}} + C = \ln y + \frac{2\delta\beta_0}{\beta_0^2+\delta^2} \arcsin \beta_0 y +$$

$$+ \frac{\delta^2}{\beta_0^2+\delta^2} \ln \left[\frac{1}{(1-y\sqrt{\beta_0^2+\delta^2})^2}\, \frac{\delta\sqrt{\beta_0^2+\delta^2}\sqrt{1-\beta_0^2 y^2+\delta^2}+\beta_0^2(1-y\sqrt{\beta_0^2+\delta^2})}{\delta\sqrt{\beta_0^2+\delta^2}\sqrt{1-\beta_0^2 y^2+\delta^2}+\beta_0^2(1+y\sqrt{\beta_0^2+\delta^2})} \right].$$

**** THE END ****

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